Previous works summary

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I am interested in the temporal stability of propagated waves, for evolution PDE. My contributions mostly study monostable fronts in one spatial dimention. These are heteroclinic waves with a linearly unstable endstate.

I also get interested into stability of constant states, periodic waves, discrete waves. Recently, I look at the long-range interactions between waves. Most of the time, I look at parabolic equations (reaction diffusion systems), although I also considered wave equations and Burgers' like equations. The tools I use originate from dynamical study of PDE: nonlinear stability derives from spectral stability.

- M. Avery and L. Garénaux, **Spectral stability of the critical front in the extended Fisher-KPP equation**, *Z. Angew. Math. Phys.* ***74****, 71 (2023).* `published`

  This article study the extended FKPP equation

  $$u_t = -\delta^2 u_{xxxx} + u_{xx} + u(1-u), \qquad t > 0, \quad x \in \mathbb{R}, \quad u(t,x) \in \mathbb{R}. \tag{1}$$

  We show that $\delta = 0$ invasion fronts persists when $\delta \neq 0$ is small, and obtain their spectral stability. This result combine with [AS21] and leads to asymptotic stability of these waves. The main obstacles dealt with in this article are: the singular limit $\delta \to 0$; the contact between essential spectrum and the imaginary axis ; the lack of comparison principle. We compose by a regularizing operator (preconditioner) and construct a local Evans function (with respect to the spectral parameter and to δ) to ensure that no eigenvalue emerge from the essential spectrum.

- L. Garénaux, **Nonlinear convective stability of a critical pulled front undergoing a Turing bifurcation at its back a case study**, *SIAM J. Math. Anal.* ***56****, 3 (2024).* `published`

  This article looks at a wave connecting two linearly unstable constant states. Both instabilities are caused by the essential spectrum: Behind the wave a Turing bifurcation occurs. Temporal stability of this wave is shown in a suitable topology (perturbations in exponentially weighted spaces both in front and behind the wave). The main technical difficulty is the necessity to use spatially unbounded weights. It is solved through an elaborate argument that requires precise estimate of the resolvent kernel as well as a the reduction to a Ginzburg-Landau amplitude equation. The latter guaranties some stability for the state behind the wave.

- L. Garénaux and L. M. Rodrigues, **Convective stability in balance laws**, *Differential Integral Equations* ***38**, 1-2 (2025).* `published`

This article study scalar balance laws

$$u_t + \partial_x(f(u)) = g(u), \qquad t > 0, \quad x \in \mathbb{R}, \quad u(t,x) \in \mathbb{R}.$$

We find smooth invasion fronts, and show their asymptotic stability in optimally weighted spaces. Furthermore, we prove an orbital stability result with respect to phase shifts. Such a result is knew, and unknown for reaction-diffusion systems. To handle quasi-linearity, we use a non-autonomous formulation of the problem. Main task is then to construct exponential weights that are time-uniform, and show resolvent estimates. In this context, we discover a slowest speed (similarly as for reaction-diffusion equations) although it can not be reached. In a second part, we look at discontinuous waves, obtained as concatenation of two smooth ones. Some of these solutions come as a two phase shift family. Finally, we consider the multidimensional problem with non-planar waves. These waves are classified according to their spectral and nonlinear stability, from a list of simple criteria, following [DR20, DR22].

- L. Garénaux and H. J. Hupkes, **Existence of monostable fronts for a KPP infinite-difference numerical scheme**, *Discrete Contin. Dyn. Syst.* ***48**, 30-47 (2026).* `published`

This article consider the space-discretized KPP equation

$$u_j'(t) = \frac{u_{j+1} - 2u_j + u_{j-1}}{h^2} + u_j(1 - u_j), \qquad t \geq 0, \quad j \in \mathbb{Z}, \quad u_j(t) \in \mathbb{R},$$

when the discretization parameter $h > 0$ is small. We show existence of invasion fronts, even when the above standard discretization of the Laplacian is replaced by a convolution with a symetric kernel with infinite support. In particular, we handle problems that do not satisfy the comparison principle. We propagate the continuous operator invertibility (when $h \to 0$) to the discrete operator (when $h > 0$), by showing h-uniform resolvent estimates. Similarly to , we use an asymptotic ansatz to cancel linear residual terms. We conclude that use of stabilizing exponential weights is possible when the convolution kernel is geometrically localized.

- L. Garénaux and B. de Rijk, **Long time behavior of small solutions in the viscoelastic Klein-Gordon equation**. `preprint`

This article is the first step to study the viscoelastic Klein-Gordon equation

$$u_{tt} - u_{xx} + u - \alpha u_{txx} = N(u), \qquad t > 0, \quad x \in \mathbb{R}, \quad u(t,x) \in \mathbb{R}. \tag{2}$$

Before looking at the above problem, we consider a simplified toy model with a single oscillating linear mode. We prove global existence of initially small solutions, with diffusive decay. The main difficulty is to control quadratic nonlinear terms, that usually leads to finite time blow-up [Fuj66]. To do so, we apply several normal form transforms [Sha85]. It is possible thanks to temporal oscillations, and allows to improve the nonlinearity exponent from two to four.

- L. Garénaux and B. de Rijk, **Global existence and decay of small solutions in a viscous half Klein–Gordon equation**. `published`

  This article complete the previous one, and deal with the full problem (2). Interactions between the two oscillating linear modes prevent to apply normal form to some cubic terms. In the terminology of [GMS09], these are both time and space resonant. To obtain existence of global in time solutions, we reduce the critical mode dynamic, and close a sharp nonlinear argument. We identify an absorption condition that the nonlinearity must satisfy.

- L. Garénaux and B. Hilder, **Linear convective stability of a front superposition with unstable connecting state**. `preprint`

  This article is a preliminary study of invasion front superposition. We construct a time dependent exponential weight that stabilize the linear dynamic. Resolvent estimate follow from a description of the numerical range, when coupling terms have low effects. Control of the residual terms is not possible in the considered weighted spaces.

- E. Bukieda, L. Garénaux and B. de Rijk, **Orbital stability of plane waves in the Klein-Gordon equation against localized perturbations**. `preprint`

  We consider planar periodic waves for the Klein-Gordon equation (2) when $\alpha = 0$. Using polar decomposition and energy estimates, we investigate stability of non-localized solutions that model the connection between two periodic waves with different phase shift. We obtain a partial orbital stability result: On every compact set, the solution is proved to converge towards a translate of the original wave. Interestingly, we are able to deal with unbounded asymptotic phase shifts.

- L. Garénaux, **Orbital stability of monostable waves for reaction-diffusion systems**. `preprint`

  This article improves known stability result for monostable diffusive fronts. Using optimal exponential weights, orbital stability of super-critical fronts with respect to translations is obtained. The result allows initial perturbations with same localization rate as the wave itself. The main difficulty is that these are not localized in weighted spaces. When they converge near the linearly unstable endstate, it is possible to extract a weak localization, and to deduce some temporal decay, following . The proof in the diffusive case relies on a Green kernel description as a translated Gaussian.

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